Advanced Studies in Pure Mathematics 42, 2004
Complex Analysis in Several Variables
pp. 115–121

A link between the asymptotic expansions of the Bergman kernel and the Szegö kernel

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Introduction

Let Ω be a strictly pseudoconvex domain in $\mathbb{C}^n$. Then the Bergman kernel K^{B} and the Szegö kernel K^{S} of Ω have singularities at the boundary diagonal. These singularities admit asymptotic expansions in powers and log of the defining function of Ω ([3], [2]) and, moreover, the coefficients of which can be expressed in terms of local invariants of the CR structure of the boundary $\partial\Omega$ as an application of the parabolic invariant theory developed in [4], [5], [1], [8], [6] and others. While these works provide a geometric algorithm of expressing the expansion of each kernel, it is not easy to read relations between them from this construction — for example, we can say very little about the relation between the log term coefficients of K^{B} and K^{S}, cf. §2.

In this note we present a method of relating these asymptotic expansions. Our strategy is to construct a meromorphic family of kernel functions K_s, $s \in \mathbb{C}$, such that K^{B} and K^{S} are realized as special values of K_s. In the case of the unit ball, $\{|z| < 1\}$, such a family is given by

$$K_s(z) = \pi^{-n}\Gamma(n-s)\,(1-|z|^2)^{s-n},$$

where $\Gamma(\alpha)$ is the gamma function, and K_{-1}, K_0 give K^{B}, K^{S}, respectively. Note that, for $s < 0$, K_s is characterized as the Bergman kernel for the weighted L^2 norm defined by the measure $(1-|z|^2)^{-s-1}/\Gamma(-s)dV$, see §1. For general strictly pseudoconvex domains, we begin by defining K_s for $s < 0$ as the weighted Bergman kernel, and then extend to $s \in \mathbb{C}$ by analytic continuation. Here we only consider the asymptotic expansion of K_s and define the analytic continuation as a meromorphic family of formal series, see §2. We then apply the invariant theory to express K_s in terms of geometric invariants of the boundary (Theorem 2). In these expansions, all K_s contain the same invariants up to universal

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Received April 15, 2002.
Partially supported by Grants-in-Aid for Scientific Research, JSPS..