Excursions of a complex analyst
into the realm of dynamical systems

Eric Bedford

§0. Introduction

The purpose of this talk is to discuss some connections between dynamics and complex analysis, especially the aspects of dynamical systems that were sufficiently interesting to me to make me drop what I was doing several years ago and enter into a long collaboration with John Smillie. One of the motivations for the work with Smillie has been to consider the dynamics of a polynomial diffeomorphism f of $\mathbf{C}^2$ which is the complexification of a map of $\mathbf{R}^2$. In general, the dynamical systems induced by f on $\mathbf{R}^2$ and $\mathbf{C}^2$ can be considerably different. However, if the complex Julia set $J \subset \mathbf{C}^2$ also happens to be a subset of $\mathbf{R}^2$, then in addition to the usual tools of real dynamics, we may also use complex methods. In the following talk, we will present the approach developed with Smillie in [BS1–5]. In §3, we outline the work [BD1,2] with Jeff Diller in which this same approach has been applied to a family of birational maps of the plane. In §4 we describe the Hénon attractor in $\mathbf{R}^2$, for which it has been difficult to actually prove anything. Although it is speculative, we present the suggestion that this phenomenon might profitably be investigated in the complex domain, a suggestion that I think Oka might have found intriguing. Given the constraints of space and time, I have limited myself to expounding a point of view and have made no attempt to survey the literature.[1]

[1]We recommend [MNTU] for an extended introduction to the dynamics of polynomial diffeomorphisms of $\mathbf{C}^2$ and Sibony [S] for a unified treatment of the iteration of rational mappings of $\mathbf{P}^k$.