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Stochastic Newton Equation with Reflecting Boundary Condition

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§1. Introduction

Let D be a bounded domain in $\mathbf{R}^d$ with a smooth boundary and $n(x)$, $x \in \partial D$, be an outer normal vector. Let $a^{ij} : \mathbf{R}^d \to \mathbf{R}$, $i, j = 1, \dots d$, be smooth functions such that $a^{ij}(x) = a^{ji}(x)$, $x \in \mathbf{R}^d$. Also, let $b^i : \mathbf{R}^{2d} \to \mathbf{R}$, $i = 1, \dots d$, be bounded measurable functions. We assume that there are positive constants C_0, C_1 such that

$$C_0|\xi|^2 \le \sum_{i,j=1}^{d} a^{ij}(x)\xi_i\xi_j \le C_1|\xi|^2, \qquad x, \xi \in \mathbf{R}^d.$$

Let L_0 be a second order linear differential operator in $\mathbf{R}^{2d}$ given by

$$L_0 = \sum_{i=1}^{d} v^i \frac{\partial}{\partial x^i} + \frac{1}{2} \sum_{i,j=1}^{d} a^{ij}(x) \frac{\partial^2}{\partial v^i \partial v^j} + \sum_{i=1}^{d} b^i(x, v) \frac{\partial}{\partial v^i}$$

Let $\tilde{W}^d = C([0,\infty); \mathbf{R}^d) \times D([0,\infty); \mathbf{R}^d)$. Now let $\Phi : \mathbf{R}^d \times \partial D \to \mathbf{R}^d$ be a smooth map satisfying the following .
(i) $\Phi(\cdot, x) : \mathbf{R}^d \to \mathbf{R}^d$ is linear for all $x \in \partial D$.
(ii) $\Phi(v, x) = v$ for any $x \in \partial D$ and $v \in T_x(\partial D)$, i.e., $\Phi(v, x) = v$ if $x \in \partial M$, $v \in \mathbf{R}^d$ and $v \cdot n(x) = 0$.
(iii) $\Phi(\Phi(v, x), x) = v$ for all $v \in \mathbf{R}^d$ and $x \in \partial D$.
(iv) $\Phi(n(x), x) \ne n(x)$ for any $x \in \partial D$.
The main theorem in the present paper is the following.

Theorem 1. *Let $(x_0, v_0) \in (\bar{D})^c \times \mathbf{R}^d$. Then there exists a unique probability measure μ over $\tilde{W}^d$ satisfying the following conditions.*
(1) $\mu(w(0) = (x_0, v_0)) = 1$.
(2) $\mu(w(t) \in D^c \times \mathbf{R}^d, t \in [0, \infty)) = 1$.

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