

Backward Regularity for some Infinite Dimensional Hypoelliptic Semi-groups

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1. *Regularity of the canonic diffusion above Virasoro algebra*
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We dedicate this work to Kiyosi Itô, the “Newton of continuous Stochastic Dynamic”, one of the most influential scholars of the last century; the topic of this paper underlines in itself the deep influence that the 1976 Kyoto Symposium [12] has had on the whole subsequent carrier of the second author who is also deeply indebted to Kiyosi Itô for fifty years of warm personal relations; his attentive support from the beginning to some of our scientific enterprises has been a key step towards their international recognition.

In classical Stochastic Analysis regularity properties are time independent : the Brownian motion is for all time Hölderian of order $(\frac{1}{2} - \epsilon)$ regular, the tangent space to the Wiener space (i.e. the Cameron-Martin space) is also time independent. The Stochastic Analysis on Loop groups have recently confirmed the paradigm that regularity properties are time independent.

It has been a surprise that regularity exponents for highly non linear infinite dimensionnal diffusion as the canonic diffusion above Virasoro algebra are time dependent [2],[9]. We shall discuss in this paper the status of tangent space to Virasoro diffusion; we shall exhibit a minimal tangent space which is time independent; it is conceivable that the maximal tangent space is time dependent, fact which will be established on a toy model. The finite dimensional root of of this phenomen lies in the fact that hypoelliptic diffusion on R^d does not satisfy simple scaling relation when the time goes to zero [4], [11].

Stability of interest models in Mathematical Finance are deeply affected by these infinite dimensional effects.