The representation theory of the Ariki-Koike and cyclotomic q-Schur algebras

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§1. Introduction

The Ariki–Koike algebras first appeared in the work of Cherednik [30] who discovered these algebras in his study of the q–analogue of Drinfeld's duality between the degenerate affine Hecke algebra and the Yangians for $\mathfrak{gl}_N$. Seven years later these algebras were rediscovered by Ariki and Koike [8] who were interested in them because they are a natural generalization of the Iwahori-Hecke algebras of types A and B. At almost the same time, Broué and Malle [21] also attached to each complex reflection group a *cyclotomic Hecke algebra* which, they conjectured, should play a role in the decomposition of the induced cuspidal representations of the finite groups of Lie type. The Ariki-Koike algebras are a special case of Broué and Malle's construction.

The deepest conjectures of Broué, Malle and Michel concerning the Ariki-Koike algebras have not yet been proved (see §2.5); however, many of the consequences of these conjectures have been established. Further, the representation theory of these algebras is beginning to be well understood. For example, the simple modules of the Ariki-Koike algebras have been classified; the blocks are known; there are analogues of Kleshchev's modular branching rules; and, in principle, the decomposition matrices of the Ariki-Koike algebras are known in characteristic zero. In many respects this theory looks much like that of the symmetric groups; in particular, there is a rich combinatorial mosaic underpinning these results which involves familiar objects like standard tableaux (indexed by multipartitions), Specht modules and so on.