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# Quantum affine algebras and crystal bases

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## §1. Introduction

The notion of a *quantum group* was introduced by Drinfel'd and Jimbo, independently, in their study of the quantum Yang-Baxter equation arising from 2-dimensional solvable lattice models ([3, 9]). Quantum groups are certain families of Hopf algebras that are deformations of universal enveloping algebras of Kac-Moody algebras. For the past 20 years, the quantum groups turned out to be the fundamental algebraic structure behind many branches of mathematics and mathematical physics.

In [18, 19, 25], Kashiwara and Lusztig independently developed the theory of *crystal bases* (or *canonical bases*) for quantum groups which provides a powerful combinatorial and geometric tool to study the representations of quantum groups. A *crystal basis* can be understood as a basis at $q = 0$ and is given a structure of colored oriented graph, called the *crystal graph*, with arrows defined by the *Kashiwara operators.* The crystal graphs have many nice combinatorial features reflecting the internal structure of integrable modules over quantum groups. In particular, they have extremely simple behavior with respect to taking the tensor product.

In this paper, we will discuss some of the recent developments in crystal basis theory for quantum affine algebras in connection with combinatorics of *Young walls* ([12, 16, 17]). In Section 2, 3 and 4, we briefly review the basic properties of Kac-Moody algebras, quantum groups and

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