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Representations of Lie algebras in positive characteristic

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About 50 years ago it was discovered that finite dimensional Lie algebras in positive characteristic only have finite dimensional irreducible representations. About 15 years ago the irreducible representations for the Lie algebra $\mathfrak{gl}_n$ were classified. About 5 years ago a conjecture was formulated that should lead to a calculation of the dimensions of these simple $\mathfrak{gl}_n$–modules if $p > n$. For Lie algebras of other reductive groups our knowledge is more restricted, but there has been some remarkable progress in this area over the last years. The purpose of this survey is to report on these developments and to update the earlier surveys [H3] and [J3].

Throughout this paper let K be an algebraically closed field of prime characteristic p. All Lie algebras over K will be assumed to be finite dimensional.

A General Theory

A.1. If $\mathfrak{g}$ is a Lie algebra over K, then we denote by $U(\mathfrak{g})$ the universal enveloping algebra of $\mathfrak{g}$ and by $Z(\mathfrak{g})$ the centre of $U(\mathfrak{g})$.

A *restricted Lie algebra* over K is a Lie algebra $\mathfrak{g}$ over K together with a map $\mathfrak{g} \to \mathfrak{g}$, $X \mapsto X^{[p]}$, often called the *p–th power map*, provided certain conditions are satisfied. The first condition says that for each $X \in \mathfrak{g}$ the element

$$\xi(X) = X^p - X^{[p]} \in U(\mathfrak{g})$$

actually belongs to the centre $Z(\mathfrak{g})$ of $U(\mathfrak{g})$. (Here X^p is the p–th power of X taken in $U(\mathfrak{g})$.) The other condition says that $\xi : \mathfrak{g} \to Z(\mathfrak{g})$ is semi-linear in the following sense: We have

$$\xi(X+Y) = \xi(X) + \xi(Y) \qquad \text{and} \qquad \xi(a\,X) = a^p\,\xi(X)$$

for all $X, Y \in \mathfrak{g}$ and $a \in K$.

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