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Zeta Functions and Functional Equations Associated with the Components of the Gelfand-Graev Representations of a Finite Reductive Group

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§0. Introduction

Zeta functions and functional equations associated with them for representations of finite groups were first discussed by Springer [18] and Macdonald [14] for certain representations over the complex field $\mathbb{C}$ of $GL_n(k)$ for a finite field $k = \mathbb{F}_q$. Their results, with one additional assumption, hold for irreducible representations over $\mathbb{C}$ of an arbitrary finite group G embedded in $GL(V)$, for an n-dimensional vector space V over k. In §1, a related functional equation is obtained for irreducible representations of Hecke algebras (or endomorphism algebras) $\mathcal{H}$ of multiplicity free induced representations of finite groups.

The functional equation 1.2.1 for an irreducible representation π of G involves an ε-factor $\varepsilon(\pi, \chi)$ which is given by

$$\varepsilon(\pi, \chi) = q^{-n^2/2}(\deg \pi)^{-1} \sum_{g \in G} \zeta_{\pi^*}(g)\chi(\mathrm{Tr}\,(g)),$$

where ζ_{π^*} is the character of the contragredient representation π^* of π, χ is a nontrivial additive character of k, and $\mathrm{Tr}\,(g)$ is the trace of g in $GL(V)$. The functional equations satisfied by irreducible representations f_π of $\mathcal{H}$, with π an irreducible component of the induced representation, have the form (see Proposition 1.5, §1)

$$f_\pi(\widetilde{h}) = \varepsilon(\pi, \chi) f_\pi(h),$$

with $h \in \mathcal{H}$, and $\widetilde{h}$ a twisted Fourier transform of h (to be defined in §1). The ε-factor $\varepsilon(\pi, \chi)$ is also given by the formula

$$\varepsilon(\pi, \chi) = f_\pi(\widetilde{e}),$$

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