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Appendix: Braiding compatibilities

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§1. Introduction

Let us recall the following basic constructions from [3]: [1] for $\mathcal{S} \in \mathrm{Perv}_{G(\widehat{\mathcal{O}})}(\mathrm{Gr})$, by taking nearby cycles we obtain $Z(\mathcal{S}) \in \mathrm{Perv}_I(\mathrm{Fl})$. Moreover, for $\mathcal{S}$ as above and $\mathcal{T} \in \mathrm{Perv}_I(\mathrm{Fl})$ we have a perverse sheaf $\mathcal{C}(\mathcal{S}, \mathcal{T}) \in \mathrm{Perv}_I(\mathrm{Fl})$ with isomorphisms

$$Z(\mathcal{S}) \star \mathcal{T} \to \mathcal{C}(\mathcal{S}, \mathcal{T}) \leftarrow \mathcal{T} \star Z(\mathcal{S}).$$

We will denote the resulting isomorphism $Z(\mathcal{S}) \star \mathcal{T} \to \mathcal{T} \star Z(\mathcal{S})$ by $u_{\mathcal{S},\mathcal{T}}$.

In addition, we will denote by $v_{\mathcal{S}_1,\mathcal{S}_2}$ the morphism $Z(\mathcal{S}_1) \star Z(\mathcal{S}_2) \to Z(\mathcal{S}_1 \star \mathcal{S}_2)$ for $\mathcal{S}_1, \mathcal{S}_2 \in \mathrm{Perv}_{G(\widehat{\mathcal{O}})}(\mathrm{Gr})$.

There are 3 properties to check:

1) Let $\mathcal{T}_1, \mathcal{T}_2$ be two I-equivariant perverse sheaves on Fl, and $\mathcal{S}$ be a $G(\widehat{\mathcal{O}})$-equivariant perverse sheaf on Gr. We must have a commutative diagram:

$$\begin{array}{ccc}
Z(\mathcal{S}) \star \mathcal{T}_1 \star \mathcal{T}_2 & \xrightarrow{u_{\mathcal{S},\mathcal{T}_1} \star \mathrm{id}_{\mathcal{T}_2}} & \mathcal{T}_1 \star Z(\mathcal{S}) \star \mathcal{T}_2 \\
{\scriptstyle u_{\mathcal{S},\mathcal{T}_1 \star \mathcal{T}_2}} \downarrow & & \downarrow {\scriptstyle \mathrm{id}_{\mathcal{T}_1} \star u_{\mathcal{S},\mathcal{T}_2}} \\
\mathcal{T}_1 \star \mathcal{T}_2 \star Z(\mathcal{S}) & \xrightarrow{\mathrm{id}} & \mathcal{T}_1 \star \mathcal{T}_2 \star Z(\mathcal{S}).
\end{array}$$

2) Let $\mathcal{S}_1, \mathcal{S}_2$ be two $G(\widehat{\mathcal{O}})$-equivariant perverse sheaves on Gr and $\mathcal{T}$–an I-equivariant perverse sheaf on Fl. We must have a commutative diagram:

$$\begin{array}{ccccc}
Z(\mathcal{S}_1) \star Z(\mathcal{S}_2) \star \mathcal{T} & \xrightarrow{\mathrm{id}_{Z(\mathcal{S}_1)} \star u_{\mathcal{S}_2,\mathcal{T}}} & Z(\mathcal{S}_1) \star \mathcal{T} \star Z(\mathcal{S}_2) & \xrightarrow{u_{\mathcal{S}_1,\mathcal{T}} \star \mathrm{id}_{Z(\mathcal{S}_2)}} & \mathcal{T} \star Z(\mathcal{S}_1) \star Z(\mathcal{S}_2) \\
{\scriptstyle v_{\mathcal{S}_1,\mathcal{S}_2} \star \mathrm{id}_{\mathcal{T}}} \downarrow & & & & \downarrow {\scriptstyle \mathrm{id}_{\mathcal{T}} \star v_{\mathcal{S}_1,\mathcal{S}_2}} \\
Z(\mathcal{S}_1 \star \mathcal{S}_2) \star \mathcal{T} & \xrightarrow{u_{\mathcal{S}_1 \star \mathcal{S}_2,\mathcal{T}}} & \mathcal{T} \star Z(\mathcal{S}_1 \star \mathcal{S}_2) & \xrightarrow{\mathrm{id}} & \mathcal{T} \star Z(\mathcal{S}_1 \star \mathcal{S}_2).
\end{array}$$

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[1] Our notations follow those of [3].