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Algebraic construction of contragradient quasi-Verma modules in positive characteristic

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§1. Introduction.

The paper grew out of attempts to fill the gap in the proof of Proposition 3.7.1 (ii) in [Ar8]. Let us recall the setting there. Denote by $\mathbf{U}_{\mathcal{A}}$ the Lusztig version of the quantum group for the root data $(Y, X, \dots)$ of the finite type $(I, \cdot)$ defined over the algebra $\mathcal{A} = \mathbb{Z}[v, v^{-1}]$ of Laurent polynomials in the variable v. Like in [L1] consider the specialization of $\mathbf{U}_{\mathcal{A}}$ in characteristic p. Namely let $\mathcal{A}'_p$ be the quotient of $\mathcal{A}$ by the ideal generated by the p-th cyclotomic polynomial. Then $\mathcal{A}'_p/(v-1)$ is isomorphic to the finite field $\mathbb{F}_p$. Thus the algebraic closure $\overline{\mathbb{F}}_p$ becomes a $\mathcal{A}$-algebra. We set $\mathbf{U}_{\overline{\mathbb{F}}_p} := \mathbf{U}_{\mathcal{A}} \otimes_{\mathcal{A}} \overline{\mathbb{F}}_p$. Let $\mathfrak{g}$ be the semisimple Lie algebra corresponding to the above root data. It is known that the quotient of the algebra $\mathbf{U}_{\overline{\mathbb{F}}_p}$ by certain central elements denoted by $\mathbf{U}_{\overline{\mathbb{F}}_p}(\mathfrak{g})$ (see [L1], the precise statement is given below, in 5.4) is isomorphic to $\mathbf{U}_{\mathbb{Z}}(\mathfrak{g}) \otimes \overline{\mathbb{F}}_p$, where $\mathbf{U}_{\mathbb{Z}}(\mathfrak{g})$ denotes the Kostant integral form for the universal enveloping algebra of $\mathfrak{g}$ (see 3.1 for the definition of $\mathbf{U}_{\mathbb{Z}}(\mathfrak{g})$).

Recall that an important step in [Ar8] consisted of constructing a certain complex of $\mathbf{U}_{\mathcal{A}}$-modules $\mathbb{D}B^\bullet_{\mathcal{A}}(\lambda)$ for a regular dominant integral weight λ called the contragradient quasi-BGG complex over $\mathbf{U}_{\mathcal{A}}$. The modules in the complex are enumerated by the Weyl group in the standard Bruhat order and are called the contragradient quasi-Verma modules. This complex appears to be a quantum analogue of a certain classical object.

Namely consider the Flag variety $\mathcal{B}_{\overline{\mathbb{F}}_p}$ for the group $G_{\overline{\mathbb{F}}_p}$ and the standard linear bundle $\mathcal{L}(\lambda)$ on it. It is known that $\mathcal{B}_{\overline{\mathbb{F}}_p}$ is stratified by $B^+_{\overline{\mathbb{F}}_p}$-orbits $\{C_{w,\overline{\mathbb{F}}_p} | w \in W\}$ called the Schubert cells. Here and below W denotes the Weyl group corresponding to the root data. Consider the

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