Advanced Studies in Pure Mathematics 39, 2004
Stochastic Analysis on Large Scale Interacting Systems
pp. 377–395

Zero-Range-Exclusion Particle Systems

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§1. Introduction

Let $\mathbf{T}_N$ denote the one-dimensional discrete torus $\mathbf{Z}/N\mathbf{Z}$ represented by $\{1, ..., N\}$. The zero-range-exclusion process that we are to introduce and study in this article is a Markov process on the state space $\mathcal{X}^N := \mathbf{Z}_+^{\mathbf{T}_N}$ ($\mathbf{Z}_+ = \{0, 1, 2, ...\}$). Denote by $\eta = (\eta_x, x \in \mathbf{T}_N)$ a generic element of $\mathcal{X}^N$, and define

$$\xi_x = \mathbf{1}(\eta_x \geq 1)$$

(namely, ξ_x equals 0 or 1 according as η_x is zero or positive). The process is regarded as a 'lattice gas' of particles having energy. The site x is occupied by a particle if $\xi_x = 1$ and vacant otherwise. Each particle has energy, represented by η_x, which takes discrete values $1, 2, \ldots$ If y is a nearest neighbor site of x and is vacant, a particle at site x jumps to y at rate $c_{\text{ex}}(\eta_x)$, where c_{ex} is a positive function of $k = 1, 2, \ldots$ Between two neighboring particles the energies are transferred unit by unit according to the same stochastic rule as that of the zero-range processes. In this article we shall give some results related to the hydrodynamic scaling limit for this model.

To give a formal definition of the infinitesimal generator of the process we introduce some notations. Let $b = (x, y)$ be an oriented bond of $\mathbf{T}_N$, namely x and y are nearest neighbor sites of $\mathbf{T}_N$, and (x, y) stands for an ordered pair of them. Define the *exclusion* operator π_b and *zero-range* operator ∇_b attached to b which act on $f \in C(\mathcal{X}^N)$ by

$$\pi_b f(\eta) = f(S_{\text{ex}}^b \eta) - f(\eta) \quad \text{and} \quad \nabla_b f(\eta) = f(S_{\text{zr}}^b \eta) - f(\eta)$$

where the transformation $S_{\text{ex}}^b : \mathcal{X}^N \mapsto \mathcal{X}^N$ is defined by

$$(S_{\text{ex}}^b \eta)_z = \begin{cases} \eta_y, & \text{if } z = x, \\ \eta_x, & \text{if } z = y, \\ \eta_z, & \text{otherwise,} \end{cases}$$

Received December 26, 2002.
Revised March 24, 2003.