Advanced Studies in Pure Mathematics 39, 2004
Stochastic Analysis on Large Scale Interacting Systems
pp. 213–232

A PDE Approach for Motion of Phase-Boundaries by a Singular Interfacial Energy

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§1. Introduction

This is a review paper on geometric motions of phase boundaries like crystal surfaces when the interfacial energy is very singular. Such motions arise in nonequilibrium problem at low temperature. Our purpose is to review a macroscopic approach describing the phenomena by a partial differential equation (PDE) with singular diffusivity. Because of nonlocal effect of singular diffusivity the notion of solution itself is unclear. In this paper we focus the problem whether a solution of approximate parabolic problem converges to a 'solution' of PDE with the singular diffusivity. We do not intend to exhaust the references.

The equilibrium of a crystal shape is often explained as a solution of an anisotropic isoperimetric problem. The problem is described as follows. Let γ be a continuous function on $\mathbf{R}^n$ which is positively homogeneous of degree one, *i.e.*, $\gamma(\lambda p) = \lambda\gamma(p)$ for all $p \in \mathbf{R}^n$, $\lambda > 0$. Assume that $\gamma(p) > 0$ for $p \neq 0$. For an oriented hypersurface S with the orientation $\mathbf{n}$ (a unit normal vector field) in $\mathbf{R}^n$ let $I(S)$ be defined by

$$I(S) = \int_S \gamma(\mathbf{n}) dS, \tag{1.1}$$

where dS denotes the surface element. The quantity $I(S)$ is called the interfacial energy and γ is called the *interfacial energy density* (depending upon the temperature τ through the structure of the crystal surface S).

Received February 4, 2003.
Revised May 8, 2003.
Partly Supported by the Grant-in-Aid for Scientific Research, No.12874024, No.14204011, the Japan Society for the Promotion of Science.