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Topological conjugacy invariants of symbolic dynamics arising from C^*-algebra K-theory

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§1 Introduction

In [Wi], R. F. Williams introduced the notions of strong shift equivalence and shift equivalence between nonnegative square matrices and showed that two topological Markov shifts are topologically conjugate if and only if the associated matrices are strong shift equivalent. He also showed that strong shift equivalence implies shift equivalence (cf. [KimR]). There is a class of subshifts called sofic subshifts that are generalized class of Markov shifts and determined by square matrices with entries in formal sums of symbols (see [Kit],[Kr4],[LM],[We],etc.). Such a square matrix is called a symbolic matrix. It is an equivalent object to a labeled graph called a λ-graph. M. Nasu in [N],[N2] generalized the notion of strong shift equivalence to symbolic matrices. He showed that two sofic subshifts are topologically conjugate if and only if their canonical symbolic matrices are strong shift equivalent ([N],[N2],see also [HN]). M. Boyle and W. Krieger in [BK] introduced the notion of shift equivalence for symbolic matrices and studied topologically conjugacy for sofic subshifts.

In [Ma6], the notions of symbolic matrix system and λ-graph system have been introduced as presentations of subshifts. They are generalized notions of symbolic matrix and λ-graph for sofic subshifts. Let Σ be a finite set. A symbolic matrix system over Σ consists of two sequences of rectangular matrices $(\mathcal{M}_{l,l+1}, I_{l,l+1}), l \in \mathbb{N}$. The matrices $\mathcal{M}_{l,l+1}$ have entries in formal sums of Σ and the matrices $I_{l,l+1}$ have entries in $\{0, 1\}$. They satisfy the following commutation relations

$$I_{l,l+1}\mathcal{M}_{l+1,l+2} = \mathcal{M}_{l,l+1}I_{l+1,l+2}, \qquad l \in \mathbb{N}.$$

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