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Quantum spin chain and Popescu systems

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§1. Introduction

In this article, we explain how Popescu systems and their dilation to representations of the Cuntz algebra are related to some problems of quantum statistical mechanics. The physics we discuss here is the quasi one-dimensional material, closely related to an unsolved problem of anti-ferromagnetic Heisenberg models. First we begin by stating our notation and the mathematical problem precisely. Our quantum spin models with an infinite degree of freedom are described as a C^*-dynamical system on a UHF C^*-algebra. The standard references of this mathematical approach are [9] and [10]. The algebra of local observables is the infinite tensor product $\mathfrak{A}_{loc}$ of the full matrix algebras. For the usual quantum system with spin s ($s = 1/2, 1, 3/2, \cdots$), the one site algebra is $M_{2s+1}(\mathbf{C})$, the set of $2s+1$ by $2s+1$ matrices, and in this case

$$\mathfrak{A}_{loc} = \bigotimes_{\mathbf{Z}} M_{2s+1}(\mathbf{C}).$$

Each component of the tensor product above is specified with a lattice site $j \in \mathbf{Z}$. The C^*-completion of $\mathfrak{A}_{loc}$ is denoted by $\mathfrak{A}$.

For any integer j and any matrix Q in $M_{2s+1}(\mathbf{C})$, $Q^{(j)}$ will be an observable Q located at the lattice site j. Thus, by $Q^{(j)}$ we denote the following element of $\mathfrak{A}$:

$$\cdots \otimes 1 \otimes 1 \otimes \underbrace{Q}_{\text{the j-th component}} \otimes 1 \otimes 1 \otimes \cdots \in \mathfrak{A}.$$

Given a subset Λ of $\mathbf{Z}$, $\mathfrak{A}_\Lambda$ is defined as the C^*-subalgebra of $\mathfrak{A}$ generated by all $Q^{(j)}$ with $Q \in M_n(\mathbf{C})$, $j \in \Lambda$. If φ is a state of $\mathfrak{A}$ the restriction of φ to $\mathfrak{A}_\Lambda$ will be denoted by φ_Λ:

$$\varphi_\Lambda = \varphi|_{\mathfrak{A}_\Lambda}.$$

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