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Finite approximations and physics over unconventional fields

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In this talk we will discuss some ideas and results from 'unconventional physics', partly from the point of view of finite approximations.

Finite approximations play an important role in many areas of mathematics. In operator algebras there are several notions of approximate finiteness, for example *hyperfinite* algebras, AF-algebras, *residually finite* algebras, to mention some.

In the context of locally compact abelian groups there is a useful notion of closeness which takes into account the Weyl structures of the groups. It was shown in [4] that – with respect to this concept of closeness – any (separable) locally compact abelian group is a limit of finite abelian groups. This notion of convergence – called convergence of Weyl systems – involves *approximation from the outside*, i.e., the approximating groups need not be subgroups of the given group.

Convergence of Weyl systems takes place at the kinematical level. The deeper problem of approximating dynamical operators requires a more detailed analysis, and was treated in [6] for the case $\mathbf{R}^n$. Here it was shown that for quantum systems with potentials of 'oscillator type' (essentially those with discrete Hamilton spectrum), the finite approximands converge to the continuous system in the strongest possible sense: eigenvalues and eigenfunctions for the finite systems converge to the corresponding objects for the continuous system. These results have later been generalized to the setting of a general locally compact group [1]. (In this general setting, though, the position and momentum operators do not have obvious interpretations.)

The above approximation results may serve as motivation for studying quantum systems over fields other than $\mathbf{R}$ and $\mathbf{C}$ – like $\mathbf{Q}_p$, for instance – since, after all, such systems, too, can be obtained as limits of finite systems (where most computations will have to take place). This is,

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