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Non-commutative Markov operators arising from subfactors

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§1. Introduction

It is well-known that there exists a close relationship between subfactor theory and (ordinary or non-commutative) probability theory. Indeed, one may observe it already in V. F. R. Jones' original paper [12], where L^1-estimate of conditional expectations plays an important role in his proof of reducibility of Jones subfactors of index larger than 4. Since then, several authors discussed the relationship between these two fields [1] [2] [8] [9] [10] [15] [16] [17] [18]. Among other notions in probability theory, the most suitable one for subfactors so far is the theory of Poisson boundaries of random walks. It is well-known that the center of the core of a subfactor can be identified with the L^∞-space of the Poisson boundary of some random walk on the principal graph.

In [11], the author obtained a precise description of the relative commutant of the fixed point subalgebra under the infinite tensor product action of the quantum group $SU_q(2)$ on the Powers factor. Indeed, it may be regarded as "the function algebra" over "the Poisson boundary" of a non-commutative Markov operator (synonymously, a unital completely positive operator) on "the group algebra" of $SU_q(2)$.

Following the same philosophy, in this note we provide a general machinery to determine the structure of the (higher) relative commutants of the core inclusions of (not necessarily strongly amenable) subfactors. These relative commutants also may be regarded as "the function algebras" of "the Poisson boundaries" of some non-commutative Markov operators of finite type I von Neumann algebras. As an easy application, we give a new proof, based on a random walk on some ladder-like graph, to the above mentioned fact about Jones inclusions.

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