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Single generation and rank of C*-algebras

Masaru Nagisa

§1. Introduction

We mainly treat a separable C*-algebra A in this article. Let S be a subset of A_{sa}. We call S a generator of A when any C*-subalgebra B of A containing S is equal to A, and we denote $A = C^*(S)$. If S is finite, then we call A finitely generated and we define the number of generators $\mathfrak{gen}(A)$ by the minimum cardinality of S which generates A. We denote $\mathfrak{gen}(A) = \infty$ unless A is finitely generated. We call a C*-algebra A singly generated if $\mathfrak{gen}(A) \leq 2$. Indeed, if $A = C^*(x, y)$ for $x, y \in A_{sa}$, then any C*-subalgebra B of A containing the element $x + \sqrt{-1}y$ is equal to A.

There are many works on single generation of operator algebras. Many of them concern to von Neumann algebras ([2],[6],[17], [19], [20], [24]). Concerning to C*-algebras, there are interesting works of D. Topping([22]), C. L. Olsen and W. R. Zame([15]). With related to them, we introduce the recent work ([11],[12]) of singly generated C*-algebras in the next section and mention the relation between singly generated C*-algebras and their ranks in the last section.

§2. Single generation of C*-algebras

Let S be a subset of a C*-algebra A satisfying $A = C^*(S)$. If A is unital, then $\{s + 2\|s\| \mid s \in S\}$ also generates A. So we may assume that an element of S is invertible. We mention about the fundamental property of $\mathfrak{gen}(\cdot)$ without the proof.

Lemma 1. [12] *Let A and B be C^*-algebras.*

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