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Direct limit decomposition for C*-algebras of minimal diffeomorphisms

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This article outlines the proof that the crossed product $C^*(\mathbf{Z}, M, h)$ of a compact smooth manifold M by a minimal diffeomorphism $h\colon M \to M$ is isomorphic to a direct limit of subhomogeneous C*-algebras belonging to a tractable class. This result is motivated by the Elliott classification program for simple nuclear C*-algebras [9], and the observation that the known classification theorems in the stably finite case mostly apply to certain kinds of direct limits of subhomogeneous C*-algebras, or at least to C*-algebras with related structural conditions. (See Section 1.) This theorem is a generalization, in a sense, of direct limit decompositions for crossed products by minimal homeomorphisms of the Cantor set (Section 2 of [32]), for the irrational rotation algebras ([10]), and for some higher dimensional noncommutative toruses ([13], [14], [24], and [5]). (In [32], only a local approximation result is stated, but the C*-algebras involved are semiprojective.) Our theorem is not a generalization in the strict sense for several reasons; see the discussion in Section 1.

There are four sections. In the first, we state the theorem and discuss some consequences and expected consequences. In the second section, we describe the basic construction in our proof, a modified Rokhlin tower, and show how recursive subhomogeneous algebras appear naturally in our context. The third section describes how to prove local approximation by recursive subhomogeneous algebras, a weak form of the main theorem. In Section 4, we give an outline of how to use the methods of Section 3 to obtain the direct limit decomposition.

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