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The ideal structure of graph algebras

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§1. Introduction

For an $n \times n$ $\{0,1\}$-matrix $A = [A(i,j)]$ without zero rows or columns the corresponding Cuntz-Krieger algebra $\mathcal{O}_A$ is defined in [4] as a C^*-algebra generated by partial isometries $\{s_i \mid i = 1, \dots, n\}$ on a Hilbert space satisfying $s_i^* s_i = \sum_{j=1}^{n} A(i,j) s_j s_j^*$. Almost from the start it was observed [25] that instead of a matrix we can use a directed graph to encode this data. It took a little bit longer though before it was realized that graphical approach may be equally successfully applied to infinite graphs. This extension (cf. [16, 15, 6, 2, 19] and references there) allows us to study by similar tools and within the same framework objects as diverse as classical Cuntz-Krieger algebras $\mathcal{O}_n$, $\mathcal{O}_\infty$, AF-algebras, and many other C^*-algebras.

A variety of methods have been employed in the investigations of graph algebras. The arguments in [16] and several subsequent papers (eg see [17]) rely heavily on the machinery of groupoids. A different approach is based on the realization of graph algebras as Cuntz-Pimsner algebras (cf. [18, 13, 7]) corresponding to suitable Hilbert bimodules over discrete abelian C^*-algebras. However it may well be that the direct approach yields the sharpest results (cf. [2, 19]).

The structure of graph algebras is fairly well-known by now. Indeed, after several earlier partial results a criterion for their simplicity has been found [21] (see also [17]). Their K-theory is readily computable [19, 23]. Their stable rank can be determined from the graph [5]. A number of other questions, like injectivity of their homomorphisms (cf. [24]) or direct sum decomposability (cf. [8]) can now be easily answered. Modelling with graph algebras has been employed in the studies of semiprojectivity (cf. [22, 20]) and pure-infiniteness (cf. [10]).

We begin this article with a brief overview of basic facts about graph algebras, illustrated with a number of examples. Then we move to our

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