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Submanifolds with Degenerate Gauss Mappings in Spheres

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§1. Introduction

Let M be a connected l-dimensional C^∞ manifold. An immersion $f\colon M \to S^n$ to the sphere (resp. $f\colon M \to \mathbb{RP}^n$ to the projective space) is called *tangentially degenerate* (or, *developable*, or, *strongly parabolic*) if its Gauss mapping $\gamma\colon M \to G_{l+1}(\mathbb{R}^{n+1})$ has rank $< l$. Here $G_{l+1}(\mathbb{R}^{n+1})$ denotes the Grassmannian of $(l+1)$-dimensional linear subspaces in $\mathbb{R}^{n+1}$. A submanifold of S^n or $\mathbb{RP}^n$ is called *tangentially degenerate* (or, *developable*, or, *strongly parabolic*) if so is the inclusion.

In the present paper we construct new examples of tangentially degenerate compact submanifolds satisfying the equality for the inequality proved by Ferus [19]. Remark that, if we have a tangentially degenerate immersed submanifold in S^n then, via the canonical double covering $\pi\colon S^n \to \mathbb{RP}^n$, we have a tangentially degenerate immersed submanifold in $\mathbb{RP}^n$.

Remark also that the notion of tangential degeneracy is invariant under the projective transformations. Recall that $\mathbb{RP}^n = G_1(\mathbb{R}^{n+1})$ and $S^n = \widetilde{G}_1(\mathbb{R}^{n+1})$ (oriented Grassmannian) have natural projective structures, respectively. In fact, M. A. Akivis clearly stated in [3] and [4] that the study of tangentially degenerate submanifolds belongs to projective geometry. Then our standpoint is as follows: We do not need the metric structures on them for the formulation of the results, while, for the proofs of the results, we use freely the metric structures.

Let M^l be compact and connected, and $f\colon M \to S^n$ a tangentially degenerate immersion. Denote by r the maximal rank of the Gauss

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