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Log C^∞-Functions and Degenerations of Hodge Structures

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Introduction

In [27], J.H.M. Steenbrink studied degenerations of Hodge structures. For $f\colon X \longrightarrow \Delta = \{z \in \mathbb{C}\ ;\ |z| < 1\}$ projective and of semi-stable degeneration, he showed that a "limit Hodge structure" appears as the limit of the Hodge structures $H^m(X_t, \mathbb{Z})$ $(m \in \mathbb{Z},\ t \in \Delta - \{0\})$. In log Hodge theory, as in [23], his theory is interpreted in the form "the higher direct images on Δ of $\mathbb{Z}_X$ carry the natural variations of polarized log Hodge structure."

In this paper, we will generalize the theory of Steenbrink in this form to the theory with coefficients (that is, we will start with general variations of polarized log Hodge structure $\mathcal{H}_\mathbb{Z}$ on X instead of $\mathbb{Z}_X$).

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