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Independence of ℓ for Intersection Cohomology (after Gabber)

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In his lecture in the conference Algebraic Geometry 2000, O. Gabber explained his proof of the following Theorem 1 that the intersection cohomology of a proper scheme is independent of ℓ.

Theorem 1. *Let X be a proper equidimensional scheme over a finite field $\mathbf{F}_q$ and let $i \in \mathbf{Z}$. For a prime $\ell \not| q$, let $IH^i(X_{\bar{k}}, \overline{\mathbf{Q}}_\ell)$ be the intersection cohomology of degree i. Then* $\det(1 - t\,\mathrm{Fr}, IH^i(X_{\bar{k}}, \overline{\mathbf{Q}}_\ell))$ *is with coefficients in $\mathbf{Z}$ and independent of $\ell \not| q$.*

The aim of this note is to give the proof of Gabber. Some details of the proofs are filled by the author, and he takes the full responsibility for the inaccuracies that may appear in this note. In the talk, Gabber also presented the proofs of other independence of ℓ results which are not contained in this short article.

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§1. Independence for $K(X)$

1.1. Notation

We work over $k = \mathbf{F}_q$. For a prime $\ell \not| q$, choose an algebraic closure $\overline{\mathbf{Q}}_\ell$. For a scheme X separated of finite type over k, $D^b_c(X, \overline{\mathbf{Q}}_\ell)$ denotes the derived category of $\overline{\mathbf{Q}}_\ell$-sheaves defined in Weil II ([De 4]). This notion of derived category is stable under the six operations $f_!$, f_*, f^*, $f^!$, $\otimes$, $R\,Hom$, and also by the Grothendieck-Verdier dualizing functor $D = D_X$ which we normalize by

$$D_X K = R\,Hom(K,\ f^!\overline{\mathbf{Q}}_\ell) \quad \text{for } f : X \to \operatorname{Spec} k.$$

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