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Bases of Chambers of Linear Coxeter Groups

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§1. Introduction

Let V be a vector space over the real numbers $\mathbb{R}$. The subgroups of $\mathrm{GL}(V)$ that are generated by reflections are called *reflection groups.* We study in this paper those reflection groups from which a polyhedral cone may be constructed and which lead to a chamber system in V. Using a result of J. Tits [5], it follows that these groups are obtained from representations of Coxeter groups. So they are called *linear Coxeter groups.* From this point of view, these groups were also extensively studied by E.B. Vinberg [6] in the case where they have a finite number of canonical generators. We extend this theory in order to investigate the reflection subgroups of a linear Coxeter group. We make no restriction on the number of generators or on the dimension of V. Our object is to present this subject using the concrete geometric methods that are associated with the chamber systems in a real vector space.

We apply these results to give a proof that a reflection subgroup of a linear Coxeter group is again a linear Coxeter group. This generalizes the result that asserts that a reflection subgroup of a Coxeter group is a Coxeter group which was independently proved by M. Dyer [3] and V.V. Deodhar [2]. Our results also characterize a base for the reflection subgroup, which will be useful in a sequel to this paper.

§2. Linear Coxeter Groups

2.1. Polyhedral Cones

Let V be a vector space over $\mathbb{R}$, and denote its dual by $V^\vee$. Let T be a subset of V. We are interested in reflection groups that act on T. Commonly the choice for T will be V itself, but in dealing with reflection subgroups, it is useful to choose T to be the convex set that

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