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On the structure of special rank one groups

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§1. Introduction

A group X generated by two different nilpotent subgroups A and B satisfying:

$(*)$ For each $a \in A^{\#}$ there exists a $b \in B^{\#}$ satisfying $A^b = B^a$ and vice versa

is called a rank one group. The conjugates of A (and B) are called the *unipotent subgroup* of the rank one group X and the conjugates of $H = N_X(A) \cap N_X(B)$ will be called the *diagonal subgroups.* If A is abelian X is called a rank one group with *abelian unipotent* subgroups, abbreviated AUS. Moreover, if for each $a \in A^{\#}$ and $b \in B^{\#}$ which satisfy $(*)$ above, also

$$(**) \qquad a^b = b^{-a} (= (b^{-1})^a)$$

holds, X is called a *special rank one group.*

Rank one groups with abelian unipotent subgroups played a fundamental role in the theory of "abstract root subgroups" [Ti1]. Indeed by (3.18)(3) and (4.15) of [Ti1] all rank one Σ-subgroups occurring in a group generated by a class Σ of abstract root subgroups of "higher rank" are special. A theory of arbitrary rank one groups was developed in §2 of [Ti2]. In both papers one is not able to say very much about the structure of rank one groups, but one has to live with properties of such groups.

By Proposition (2.1) of [Ti2] the following are equivalent:

(i) $X = \langle A, B \rangle$ is a rank one group.

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