Advanced Studies in Pure Mathematics 32, 2001
Groups and Combinatorics — in memory of Michio Suzuki
pp. 379–390

The Shape of the Classification of the Finite Simple Groups

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This is a general survey of the Classification of the Finite Simple Groups with particular emphasis on the current project of Gorenstein, Lyons and Solomon (GLS) directed towards the revision of a substantial segment of the Classification proof.

There are two principal strategies at present directed towards a Classification proof. The one employed in the first successful proof and also, with certain modifications, in the GLS proof, I shall refer to as the Semisimple Approach to the Classification. The other, which has been the object of considerable activity recently, I shall refer to as the Unipotent Approach to the Classification. Each has its advantages and its drawbacks and neither is, at present, completely independent of the other. In unison they provide a complete proof of the Classification Theorem. A question at present is the natural domain for each of these methods. Of course the future may bring entirely new and wonderful approaches to the subject.

The modern history of the Classification began around 1950 when several mathematicians – notably Brauer, Suzuki and Wall – began to investigate simple groups of even order satisfying certain local conditions. This work eventually congealed into the Brauer-Suzuki-Wall Theorem [BSW] characterizing the two-dimensional projective special linear groups over finite fields. Brauer in particular championed the strategy of characterizing finite simple groups of even order by the centralizer of an involution. Suzuki, on the other hand, established the nonexistence of finite simple CA-groups of odd order [S1]. (A group G is a CA-group if the centralizer of every nonidentity element of G is abelian.) This result was the inspiration for the Feit-Thompson Theorem proving the nonexistence of nonabelian finite simple groups of odd order.

Meanwhile Suzuki pursued the classification of transitive permutation groups of odd degree in which the stabilizer of a point has a regular

Received May 27, 1999.
Revised June 16, 2000.