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On the vertices of modules in the Auslander–Reiten quiver III

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§0. Introduction

Let kG be the group algebra of a finite group G over a field k of characteristic p, where p is a prime. We denote the stable Auslander–Reiten quiver (AR quiver for short) of kG by $\Gamma_s(kG)$. For the definition of an AR quiver, see [B]. It is known that each connected component Γ of $\Gamma_s(kG)$ has the uniquely determined tree class $\mathcal{T}$. The AR component Γ is isomorphic as graphs to $\mathbf{Z}\mathcal{T}/\pi$, where $\mathbf{Z}\mathcal{T}$ is the graph obtained in a standard way from countably many copies of the tree $\mathcal{T}$ and π is a certain subgroup of $Aut(\mathbf{Z}\mathcal{T})$. Since the important paper by Webb [W] was published, many results concerning the tree classes have been obtained. (See [Be], [E3], [E4], [ES] and [O1].) In the present paper, assuming that k is a perfect field, we determine all the tree classes, not the possibilities of them, completely. The following should be the final result in this nature.

Theorem A. *Let k be a perfect field. Then the tree class of a connected component of $\Gamma_s(kG)$ is one of the following: A_n, $\tilde{A}_{1,2}$, A_∞, $\tilde{B}_3$, B_∞, D_∞, or A_∞^∞. Moreover, each of the above in fact occurs. Furthermore, the following hold. Here D is a defect group of the block to which the modules in Γ belong.*

(i) *B_∞ occurs only when D is dihedral.*
(ii) *D_∞ occurs only when D is semidihedral.* ([E3], [E4])
(iii) *A_∞^∞ occurs only when D is dihedral or semidihedral.* ([E3], [E4])
(iv) *$\tilde{A}_{1,2}$ or $\tilde{B}_3$ occurs only when D is a four group.* ([Be], [ES])

For the notation of the tree classes, we follow 2.30 of [B]. In particular,

$$\tilde{A}_{1,2} : \cdot \xrightarrow{(2,2)} \cdot, \quad \tilde{B}_3 : \cdot \xrightarrow{(1,2)} \cdot \longrightarrow \cdot \xrightarrow{(2,1)} \cdot, \quad B_\infty : \cdot \xrightarrow{(1,2)} \cdot \longrightarrow \cdot \longrightarrow \cdots .$$

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