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A Remark on the Loewy Structure for the Three Dimensional Projective Special Unitary Groups in Characteristic 3

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§1. Introduction and Notation

The purpose of this note is to give an alternative and easier proof of a recent result by K. Hicks [6, Theorem 1.1], which was on the Loewy and socle structure of the projective indecomposable modules in the principal 3-block of the projective special unitary group $\mathrm{PSU}_3(q^2) = \mathrm{U}_3(q)$ for a power q of a prime satisfying $q \equiv 2$ or $5 \pmod 9$ over an algebraically closed field of characteristic 3. In her paper K. Hicks used so-called Auslander-Reiten theory on representations of artin algebras (see [1]). Actually, in her paper [6], the key tool was a result, which was due to K. Erdmann [4] and S. Kawata [8] on Auslander-Reiten quivers of type A_∞ for group algebras of finite groups. On the other hand, our proof does not need the Auslander-Reiten theory (except a result due to P. Webb [15]) but just well-known results on modular representation theory of finite groups.

We use the following notation and terminology. Throughout this paper, k is always an algebraically closed field of characterictic $p > 0$, and G is always a finite group. For an element $g \in G$ we denote by $|g|$ the order of g. For a power q of a prime, $\mathbb{F}_q$ is the field of q elements, and we use the notation $\mathrm{GL}_n(q)$, $\mathrm{SL}_n(q)$, $\mathrm{PGL}_n(q)$, $\mathrm{PGU}_n(q)$, $\mathrm{PSU}_n(q)$ for a positive integer n in a standard fashion (see [7]). We denote by C_n the cyclic group of order n for a positive integer n. Let A be a finite-dimensional k-algebra. Then, $A^\times$ denotes the set of all units (invertible elements) in A, and $J(A)$ denotes the Jacobson radical of A. In this paper *modules* mean always finitely generated right modules, unless stated otherwise. Let M be an A-module. We denote by $\mathrm{Soc}(M)$ and $P(M)$ the socle of M and the projective cover of M, respectively.

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