Advanced Studies in Pure Mathematics 32, 2001
Groups and Combinatorics — in memory of Michio Suzuki
pp. 325–336

The calculation of the character of Moonshine VOA

Takeshi Kondo

§1. Introduction

In Miyamoto [M3], [M6] and Dong-Griess-Höhn [DGH], they described the structure of the Moonshine VOA $\mathbf{V}^\natural$ by using two binary codes $D^\natural, S^\natural$ and Ising models $L(\frac{1}{2},0),\ L(\frac{1}{2},\frac{1}{2}),\ L(\frac{1}{2},\frac{1}{16})$.

The purpose of this note is to calculate the character of $\mathbf{V}^\natural$ and the Thompson series of two involutions of Monster $\mathrm{Aut}(\mathbf{V}^\natural)$ ($2A, 2B$-involutions of Monster) explicitly by following the descriptions of $\mathbf{V}^\natural$ in [M3],[M6] and [DGH]. As is well known (cf.[CN]), these are equal to

$$j(z)-744,\ \left(\frac{\eta(z)}{\eta(2z)}\right)^{24}+2^{12}\left(\frac{\eta(2z)}{\eta(z)}\right)^{24}+24,\ \left(\frac{\eta(z)}{\eta(2z)}\right)^{24}+24$$

respectively, where $j(z)$ is the well known elliptic modular function and $\eta(z)$ is Dedekind's η-function. Also see a remark at the end of §4 for the calculations of Thompson series for some other elements. Finally, in §5, we will mention a little bit about VOA of "Reed Müller type".

§2. Ising models

2.1. Virasoro Algebra

An infinite dimensional Lie algebra **Vir** having a basis $\{L(m)\ (m \in \mathbf{Z}),\ \mathbf{c}\}$ is called Virasoro algebra if they satisfies

$$[L(m),\mathbf{c}]=0,\ [L(m),L(n)]=(m-n)L(m+n)+\frac{m^3-m}{12}\delta_{m+n,0}\mathbf{c}.$$

Let $L(c,h)$ be an irreducible module of **Vir** with central charge $c\ (\in \mathbf{C})$ and highest weight $h\ (\in \mathbf{C})$. Namely, there exists a vector $v \in$

Received August 31, 1999.
Revised May 16, 2000.