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Some Results on Modular Forms — Subgroups of the Modular Group Whose Ring of Modular Forms is a Polynomial Ring

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§1. Introduction

This paper is the first of the sequel of papers on the joint work of these authors on modular forms. We consider the problem of determining finite index subgroups of the modular group $\mathrm{SL}(2,\mathbb{Z})$ whose ring of modular forms is isomorphic to a polynomial ring. First, in this paper, we consider this question for modular forms of integral weights. In subsequent papers, we will consider the problem for modular forms of half-integral weights, and more generally, of $1/l$-integral weights. It turns out that the case of $l = 5$ is particularly interesting in connection with the classical work of F. Klein [9], as well as its analogy with the other two cases of $l = 1$ and $l = 2$, which are related to ternary and binary self-dual codes, respectively. In this first paper, we explain our overall motivation, and we prove the results only for the integral weight case. We remark that some preliminary announcements of some of the results given in the present paper have been made in two unofficial publications [2] and [17] written in Japanese.

§2. Statement of Results

Let Γ be a finite index subgroup of $\mathrm{SL}(2,\mathbb{Z})$. We denote by $\mathfrak{M}(\Gamma)$ the ring of modular forms of integral weights on the group Γ. It is well known that

$$\mathfrak{M}(\mathrm{SL}(2,\mathbb{Z})) = \mathbb{C}[E_4, E_6], \tag{1}$$

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