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A characterization of $^2E_6(2)$

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§1. Introduction

This paper is part of a program to provide a uniform, self-contained treatment of part of the foundations of the theory of the sporadic finite simple groups. More precisely our eventual aim is to provide complete proofs of the existence and uniqueness of the twenty-six sporadic groups and to derive the basic structure of each sporadic. The two books [SG] and [3T] make a beginning on that program.

In this paper we provide a uniqueness proof for the group $^2E_6(2)$. Of course $^2E_6(2)$ is a group of Lie type, not a sporadic group, but in order to treat the Monster and the Baby Monster, one first needs to treat $^2E_6(2)$. Thus this paper begins that part of the program dealing with the large sporadics.

Suzuki was one of the pioneers in identifying finite groups from information on subgroup structure. His characterization of $L_3(2^n)$ in [S] identifies those groups by producing a BN-pair. That approach is not so different from the one adopted in our program. Indeed in the work of S. Smith and the author on quasithin groups, the groups $L_3(2^n)$, n even, can not quite be handled using our standard methods, so we appropriate a clever counting argument of Suzuki's from [S] to fill the gap. Hopefully Suzuki would regard this paper as continuing a tradition which he pioneered.

Define a finite group G to be of *type* $^2E_6(2)$ if G possesses an involution z such that $F^*(C_G(z)) = O_2(C_G(z))$ is extraspecial of width 10, $C_G(z)/O_2(C_G(z)) \cong U_6(2)$, and z not weakly closed in $O_2(C_G(z))$ with respect to G.

Define G to be of *type* $\mathbf{Z}_2/^2E_6(2)$ if G possesses an involution z such that $F^*(C_G(z)) = O_2(C_G(z))$ is extraspecial of width 10 and $C_G(z)$ has a subgroup H of index 2 such that $H/O_2(C_G(z)) \cong U_6(2)$, and z is not weakly closed in $O_2(C_G(z))$ with respect to G.

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