Advanced Studies in Pure Mathematics 31, 2001
Taniguchi Conference on Mathematics Nara '98
pp. 265–286

Large Deviation and Hydrodynamic Scaling

Srinivasa R. S. Varadhan

§1. What are Large Deviations?

The theory of large deviations is, roughly speaking, a method of describing the rapidity with which probability distributions depending on a parameter approach the degenerate distribution at some point as the parameter becomes large.

Let us suppose that for each n there is a probabilty measure P_n on some space Ω_n defined on some σ-field Σ_n. There is a complete separable metric space X with its Borel sets $\mathcal{B}$, such that for each n there is a measurable map Φ_n of Ω_n into X. We denote the induced measure $P_n\Phi_n^{-1}$ on $(X, \mathcal{B})$ by Q_n. Actually it is the situation $(X, \mathcal{B}, Q_n)$ that will be of interest to us. As $n \to \infty$ the measures Q_n will converge weakly to a limit which will be degenrate at some point x_0 of X. This is usually a 'law of large numbers', statement. In particular for any closed set $A \subset X$, with $x_0 \notin A$,

$$\lim_{n\to\infty} Q_n(A) = 0. \tag{1.1}$$

If the parametrization has been chosen properly, the convergence in the limit (1.1) will often be exponentially fast and

$$\lim_{n\to\infty} \frac{1}{n} \log Q_n(A) = -\Psi(A) \tag{1.2}$$

will exist atleast for a large class nice sets. Since the exponential behavior of a sum is the same as that of the larger of the summands

$$\Psi(A \cup B) = \min\{\Psi(A), \Psi(B)\}$$

and one can expect $\Psi(A)$ to be given by a formula of the type

$$\Psi(A) = \inf_{x\in A} I(x)$$

Received December 2, 1999.