

The Intrinsic Hodge Theory of p -adic Hyperbolic Curves

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§1. Uniformization Theory as a Hodge Theory at Arithmetic Primes

(A) Uniformization as a Catalogue of Rational Points

We begin our discussion by posing the following fundamental problem concerning algebraic varieties over the complex numbers (where, roughly speaking, an “algebraic variety over the complex numbers” is a geometric object defined by polynomial equations with coefficients which are complex numbers):

Problem: Given an *algebraic variety* Z over \mathbf{C} , it is possible to give some sort of *natural explicit catalogue* of the rational points $Z(\mathbf{C})$ of Z ?