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# Probability and Geometry

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**Summary**

**PART I : Finite dimensional Geometry.**

1. Passage from the local to the global through Comparison Lemmas for SDE.
2. The ground state of a vector bundle.
3. Transfer principle from ODE to SDE and Stochastic Calculus of Variations.
4. Mean Value Formulae for harmonic differential forms.

**PART II : Geometry of Path Spaces.**

5. Path Space as a parallelized manifold; Tangent Process.
6. Intertwining formulae; Integration by part formulae.
7. Construction of measures in infinite dimension through infinitesimal geometry.

**Part III : Differential Geometry in infinite dimension and Stochastic Analysis.**

8. Vectors fields; their divergence and their flow.
9. Divergence and Geometrization of Anticipative Stochastic Calculus.
10. Geometric measure theory, Principle of Descent.

**PART IV : Integration on some infinite dimensional groups.**

11. From Path Group to Loop Group.
12. Heat equation on some infinite dimensional groups.

Fifty years back, Kiyosi Itô started the theory of *Stochastic Differential Equations* (SDE) in a fully geometric framework; in particular he constructed the heat process associated to an elliptic operator on a manifold by patching together Stochastic Differential Equations, the change

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