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The Topology of Real and Complex Algebraic Varieties

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Contents

§1. Introduction

Definition 1.1. Let $f_i(x_1, \dots, x_n)$ be polynomials whose coefficients are real or complex numbers. An *affine algebraic variety* is the common zero set of finitely many such polynomials

$$X = X(f_1, \dots, f_k) := \{\mathbf{x} \mid f_i(\mathbf{x}) = 0, \forall i\}.$$

To be precise, I also have to specify where the variables x_i are. If the f_i have complex coefficients then the only sensible thing is to let the x_i be complex. The resulting topological space is

$$X(\mathbb{C}) := \{\mathbf{x} \in \mathbb{C}^n \mid f_i(\mathbf{x}) = 0, \forall i\},$$

which we always view with its Euclidean topology. If the f_i have real coefficients then we can let the x_i be real or complex. Thus we obtain two "incarnations" of a variety

$$X(\mathbb{R}) := \{\mathbf{x} \in \mathbb{R}^n \mid f_i(\mathbf{x}) = 0, \forall i\}, \quad \text{and}$$
$$X(\mathbb{C}) := \{\mathbf{x} \in \mathbb{C}^n \mid f_i(\mathbf{x}) = 0, \forall i\}.$$

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