Advanced Studies in Pure Mathematics 31, 2001
Taniguchi Conference on Mathematics Nara '98
pp. 51–74

Quantum Vertex Algebras

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§1. Introduction

The purpose of this paper is to make the theory of vertex algebras trivial. We do this by setting up some categorical machinery so that vertex algebras are just "singular commutative rings" in a certain category. This makes it easy to construct many examples of vertex algebras, in particular by using an analogue of the construction of a twisted group ring from a bicharacter of a group. We also define quantum vertex algebras as singular braided rings in the same category and construct some examples of them. The constructions work just as well for higher dimensional analogues of vertex algebras, which have the same relation to higher dimensional quantum field theories that vertex algebras have to one dimensional quantum field theories.

One way of thinking about vertex algebras is to regard them as commutative rings with some sort of singularities in their multiplication. In algebraic geometry there are two sorts of morphisms: regular maps that are defined everywhere, and rational maps that are not defined everywhere. It is useful to think of a commutative ring R as having a regular multiplication map from $R \times R$ to R, while vertex algebras only have some sort of rational or singular multiplication map from $R \times R$ to

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Received March 8, 1999.

Supported by a Royal Society professorship. This paper was written at the Max-Planck institute in Bonn.