

Adele Geometry of Numbers

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Dedicated to Professors Ichiro Satake and Takashi Ono

This is a historical and expository account on adèle geometry. The word *adèle geometry* appeared in the lectures ([W4], 1959–1960) by A. Weil which were stimulated by works of Siegel–Tamagawa on quadratic forms. Our main concern here is to exhibit the strings of thoughts in the development of this topic originated from Minkowski’s *geometry of numbers*. The subject was originally related to integral geometry and some diophantine problems, and we discuss such aspects in adèle geometry on homogeneous spaces.

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