Advanced Studies in Pure Mathematics 30, 2001
Class Field Theory – Its Centenary and Prospect
pp. 387–399

Iwasawa Invariants of $\mathbb{Z}_p$-Extensions over an Imaginary Quadratic Field

Manabu Ozaki

§1. Introduction

Let k be a number field and $p \geq 2$ a prime number. For a $\mathbb{Z}_p$-extension K/k we denote by $\lambda(K/k)$ and $\mu(K/k)$ the Iwasawa λ- and μ-invariants, respectively. If k is not totally real, k has infinitely many different $\mathbb{Z}_p$-extensions. We therefore are interested in the behavior of $\lambda(K/k)$ and $\mu(K/k)$ as K varies over all $\mathbb{Z}_p$-extension fields over the number field k. Greenberg initiated the study of this problem in [4], and obtained some results on the behavior of $\lambda(K/k)$ and $\mu(K/k)$. For example he proved the boundedness of $\mu(K/k)$ for fixed k and p under some assumption on the base field k and the prime p. After Greenberg's work, Babaĭcev and Monsky independently established the boundedness of $\mu(K/k)$ without any assumption ([1], [12]).

The behavior of λ-invariants is more difficult to study than that of μ-invariants. In the present paper, we shall investigate the case where the base field is an imaginary quadratic field, and give the following theorem:

Theorem 1. *Let k be an imaginary quadratic field and $p \geq 2$ a prime number. Assume that the prime p splits in k and the class number of k is prime to p. Then $\lambda(K/k) = 1$ and $\mu(K/k) = 0$ for all but finitely many $\mathbb{Z}_p$-extensions K over k.*

We shall make some remarks on the theorem.
(1) If p does not split in a number field F and the class number of F is prime to p, then $\lambda(K/F) = \mu(K/F) = \nu(K/F) = 0$ for every $\mathbb{Z}_p$-extension K/F by Iwasawa's result ([6]). Hence only the case where p splits in the imaginary quadratic field k is interesting under the assumption that p does not divide the class number of k.

Received September 4, 1998.
Revised January 11, 1999.