

Nonabelian Local Reciprocity Maps

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There are several approaches to construct the reciprocity map, the essence of class field theory, which links the maximal abelian quotient (or sometimes the maximal abelian pro- p -quotient) of the absolute Galois group of a particular field with an appropriate abelian object associated to the field such that certain functorial properties hold.

One of those approaches originates from works of Dwork, Serre, Hazewinkel [D, S, H1, H2], Iwasawa [I1, I2] and Neukirch [N1, N2]. Recall it briefly.

Let F be a local field with finite residue field. Let F^{ur} be the maximal unramified extension of F and let \widehat{F} be the completion of F^{ur} . Let L be a finite Galois extension of F and $L^{\text{ur}} = LF^{\text{ur}}$, $\widehat{L} = L\widehat{F}$.

For an element σ of $\text{Gal}(L/F)$ let $\tilde{\sigma}$ be any element of $\text{Gal}(L^{\text{ur}}/F^{\text{ur}})$ such that $\tilde{\sigma}|_L = \sigma$ and $\tilde{\sigma}|_{F^{\text{ur}}}$ is a positive integer power of the Frobenius automorphism $\varphi \in \text{Gal}(F^{\text{ur}}/F)$. Let Σ be the fixed field of $\tilde{\sigma}$; it is a finite extension of F .

Let $\text{Gal}(L/F)^{\text{ab}}$ be the maximal abelian quotient of $\text{Gal}(L/F)$.

Define the map [N1, N2]

$$\mathbf{N}: \text{Gal}(L/F) \rightarrow F^*/N_{L/F}L^*$$

by $\sigma \rightarrow N_{\Sigma/F}\pi_{\Sigma} \bmod N_{L/F}L^*$ where π_{Σ} is any prime element of Σ . During the conference on class field theory in Tokyo, June 1998, Professor Tsuneo Tamagawa informed the author that similar constructions were independently developed by Iwasawa. We call \mathbf{N} the Neukirch–Iwasawa map.

Received August 29, 1998

Revised February 9, 1999

This work was partially done during my visit to Japan in 1998. I would like to express my gratitude to the Royal Society, RIMS, JSPS and JAMS for support, to the organizers of the conference of class field theory, and to Professor Masato Kurihara for great hospitality during my stay in Tokyo Metropolitan University in June 1998.