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Geometry of complex surface singularities

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§ Introduction.

In the local study of complex analytic spaces, it is natural to investigate the behaviour of the tangent spaces near a singular point. In the general case of equidimensional singularities, after choosing a local embedding of the singular space into a complex affine space, B. Teissier and the author have given the structure of the limit of tangent hyperplanes, i.e. hyperplanes containing a tangent space at a non-singular point, in terms of a family of cones contained in the tangent cone of the singularity and called the Auréole of the singularity (see [LT2]).

In the case of surface singularities, the Auréole is given by the tangent cone and a finite number of generatrices of the tangent cone called the exceptional tangents. Recent works of J. Snoussi showed that these exceptional tangents coincide with the special generatrices of Gonzalez and Lejeune ([GL]). His result is based on the fact that, after choosing a local embedding of the surface into a complex affine space, a hyperplane is not a limit of tangent hyperplanes if and only if its intersection with the normal surface singularity is a curve with a Milnor number (in the sense of Buchweitz and Greuel [BG]) which is minimum. This work enhances the interest in the local geometry of complex surface singularities that we began in [L3] and [LT1].

This paper is essentially a survey of results about the limits of tangent hyperplanes of a normal surface singularity. It gives a geometrical approach in the study of a normal surface singularity and suggests new research interests in effective resolutions of normal surface singularities. In particular, it should lead to effective bounds for the number of normalized blowing-up needed to solve the singularity.

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