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Assigned base conditions and geometry of foliations on the projective plane

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§0. Introduction

This paper deals with algebro-geometric properties of foliations on the projective plane. Our motivation comes from the study of some linear systems with assigned base conditions which are associated to the geometry of a foliation $\mathcal{F}$ or to its invariant curves.

First, one has the linear system of polars relative to $\mathcal{F}$ introduced in [8]. This is a net of curves (one for each $q \in \mathbf{P}^2$ passing through it) of degree $r+1$, where r is the degree of $\mathcal{F}$. The base points of the net are the singularities of $\mathcal{F}$. One also has infinitely near base points, namely, those necessary to eliminate the indeterminacy of the rational map $\Phi : \mathbf{P}^2 \longrightarrow \check{\mathbf{P}}^2$ given by the net.

In this paper we study the local and global geometry of the polar net. For a non-base point q, we prove that the pencil of polars through q has very special properties: It contains as a special member the polar P_q through q and all its members, except at most one, are smooth. The polar P_q itself is smooth and it is tangent to the line L_q assigned by $\mathcal{F}$ at q. Thus, the local invariant given by the intersection number

$$\kappa_q = I_q(P_q, L_q) \tag{0.1}$$

is such that $\kappa_q \geq 2$. The locus of points q such that $\kappa_q \geq 3$ or q is a base point is shown to be the so-called jacobian curve of the net ([12], p.115). The jacobian is a curve of degree $3r$ which is associated to $\mathcal{F}$ in this way. Finally, we study the behaviour of both the polars and the

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