Advanced Studies in Pure Mathematics 28, 2000
Combinatorial Methods in Representation Theory
pp. 103–112

Invariants for Representations of Weyl Groups, Two-sided Cells, and Modular Representations of Iwahori-Hecke Algebras

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§1. Introduction

1.1. q-Series identity.

Let $s_\lambda(x)$ be the Schur function in infinite variables $x = (x_1, x_2, \dots)$ corresponding to a Young diagram λ. For each node v in the diagram λ, $h(v)$ denotes the hook length of λ at v. Cf. [9] for the Young diagrams and related notions. In a recent work [7], Kawanaka obtained a q-series identity

$$\sum_\lambda I_\lambda(q) s_\lambda(x) = \prod_i \prod_{r=0}^{\infty} \frac{1 + x_i q^{r+1}}{1 - x_i q^r} \prod_{i<j} \frac{1}{1 - x_i x_j}, \tag{1}$$

where

$$I_\lambda(q) = \prod_{v \in \lambda} \frac{1 + q^{h(v)}}{1 - q^{h(v)}}, \tag{2}$$

and the sum on the left hand side of (1) is taken over all Young diagrams λ. If $q = 0$, then (1) reduces to the Schur-Littlewood identity.

Using (1), Kawanaka showed that for a Youndg diagram λ with n nodes, (2) is expressed as

$$I_\lambda(q) = |\mathfrak{S}_n|^{-1} \sum_{s \in \mathfrak{S}_n} \chi_\lambda(s^2) \frac{\det(1 + q\rho(s))}{\det(1 - q\rho(s))}, \tag{3}$$

where χ_λ is the irreducible character of the symmetric group $\mathfrak{S}_n$ corresponding to λ and $\rho : \mathfrak{S}_n \to GL_n(\mathbb{Z})$ is the representation of $\mathfrak{S}_n$ by permutation matrices.

Received April 1, 1999.