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Approximation Results for Kazhdan-Lusztig Polynomials

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§1. Introduction

In their fundamental paper [7] Kazhdan and Lusztig defined, for every Coxeter group W, a family of polynomials, indexed by pairs of elements of W, which have become known as the Kazhdan-Lusztig polynomials of W (see, e.g., [6], Chap. 7). These polynomials are intimately related to the Bruhat order of W and to the geometry of Schubert varieties, and have proven to be of fundamental importance in representation theory. In order to prove the existence of these polynomials Kazhdan and Lusztig used another family of polynomials (see [7], §2) which are intimately related to the multiplicative structure of the Hecke algebra associated to W. These polynomials are known as the R-polynomials of W (see, e.g., [6], §7.5) and their importance stems mainly from the fact that their knowledge is equivalent to that of the Kazhdan-Lusztig polynomials.

The main idea of this work is to use the theory of P-kernels developed by Stanley in [10] to approximate the Kazhdan-Lusztig polynomials with other "KLS-functions" (see §2 for definitions) that are easier to compute. In particular, we characterize the pairs $u, v \in W$ such that the Kazhdan-Lusztig polynomials of the subintervals of $[u, v]$ satisfy certain vanishing properties or, more generally, coincide with some given function in the incidence algebra of W, up to a given order. Two of our results generalize and refine previous ones that have appeared in [7] and [3].

The theory of P-kernels also naturally leads to define and study certain polynomials, indexed by pairs of elements of W, that are "dual"

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