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Crystal Bases for Quantum Superalgebras

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§1. Introduction

Associated with each integrable module M for the quantized enveloping algebra $U_q(\mathfrak{g})$ of a symmetrizable Kac-Moody Lie algebra $\mathfrak{g}$, there is a remarkable basis at $q = 0$, the *crystal base*, which was introduced by Kashiwara [Ka1]. If $\mathbf{A}$ denotes the local ring of all rational functions $f/g \in \mathbf{Q}(q)$ with $g(0) \neq 0$, then M contains an $\mathbf{A}$-lattice L, called the *crystal lattice.* The crystal base is a certain basis B for the $\mathbf{Q}$-vector space L/qL, which possesses many noteworthy features. It is well-behaved with respect to tensor products; it is preserved under the action of the modified root vector operators $\tilde{e}_i$ and $\tilde{f}_i$ (what are often called *Kashiwara operators*); and it has important connections with combinatorial bases of tableaux (see [MM], [KN], [KM], and [L]). Crystal bases play a prominent role in two-dimensional solvable lattice models, where the parameter q corresponds to the temperature in the lattice model. Since $q = 0$ corresponds to absolute zero temperature, one expects special behavior at this particular value, and the crystal base reflects this exceptional behavior.

In this work we describe a crystal base theory for quantum superalgebras. Basic definitions and general results on crystal bases for Kac-Moody superalgebras are presented in Sections 2, 3, and 4. Section 5 describes crystal bases for the orthosymplectic Lie superalgebra $\mathfrak{osp}(1, 2n)$, and Section 6, for affine Kac-Moody superalgebras. Sections 7, 8, and

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