Advanced Studies in Pure Mathematics 28, 2000
Combinatorial Methods in Representation Theory
pp. 1–20

Robinson-Schensted Correspondence and Left Cells

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§1. Introduction

This is based on [A]. In [A], I explained several theorems which focused on a famous theorem of [KL] that two elements of the symmetric group belong to a same left cell if and only if they share a common Q-symbol. The first half of [A] was about a direct proof of this theorem (Theorem A), and the second half was about relation between primitive ideals and left cells, and I explained another proof of this theorem.

The reason why I gave a direct proof which was different from the proof in [KL] was that the proof in [KL] was hard to read: It relied on [V1, 6], which in turn relied on [Jo1], the full paper of which is not yet available even today. Note also that the theorem itself is not stated in [KL]. But the beginning part of the proof of [KL, Theorem 1.4] gives some explanation on the relation between left cells in the sense of Kazhdan and Lusztig and Vogan's generalized τ-invariants in the theory of primitive ideals. In this picture, Theorem A is derived from Joseph's theorem.

Lack of a clear proof in the literature lead Garsia and McLarnan to the publication of [GM]. [1] The proof given in [GM] is close to [A], but the line of the proof in [GM] is interrupted with combinatorics of tableaux, which is not necessary. In fact, after we read to the fourth section of [KL], which is the section for some preliminaries to the proof of [KL, Theorem 1.4], we can give a short and elementary proof of Theorem A in a direct way, as I will show below.

I rush to say that my proof was not so original: It copied argument in [Ja1, Satz 5.25] for Joseph's theorem. This was the reason why I

Received March 8, 1999.
Revised October 21, 1999.

[1]It is worth mentioning that Garsia and McLarnan wrote in [GM] that they were benefitted by A.Björner's lecture notes and R.King's lecture notes. Both of these notes are still not available, and it seems that preliminary version of them were circulated in a very restricted group of people around the time.