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On the fundamental group of the complement of a complex hyperplane arrangement

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Dedicated to Professor Peter Orlik on his 60th birthday

§1. Introduction

Let $\mathbf{K}$ be a field, and let $V = \mathbf{K}^l$ be a finite dimensional vector space over $\mathbf{K}$. An *arrangement of hyperplanes* in V is a finite family $\mathcal{A}$ of affine hyperplanes of V. The *complement* of $\mathcal{A}$ is defined by

$$M(\mathcal{A}) = V \setminus \bigcup_{H \in \mathcal{A}} H \ .$$

If $\mathbf{K}$ is $\mathbf{C}$, then the complement $M(\mathcal{A})$ is an open and connected subset of V.

The present paper is concerned with fundamental groups of complements of complex arrangements of hyperplanes.

The most popular such a group is certainly the pure braid group; it appears as the fundamental group of the complement of the "braid arrangement" (see [OT]). So, $\pi_1(M(\mathcal{A}))$ can be considered as a generalization of the pure braid group, and one can expect to show that many properties of the pure braid group also hold for $\pi_1(M(\mathcal{A}))$. However, the only general known results on this group are presentations [Ar], [CS1], [Ra], [Sa1]. Many interesting questions remain, for example, to know whether such a group is torsion free.

We focus in this paper on two families of arrangements of hyperplanes, to the fundamental group of which many well-known results on the pure braid group can be extended. Both of them, of course, contain the braid arrangement. These families are the "simplicial arrangements" and the "supersolvable arrangements". Note that there is another well-understood family of arrangements, the "reflection arrangements" (see

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