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Recent progress of intersection theory for twisted (co)homology groups

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§1. Introduction

Maybe you have ever seen at least one of the following formulae:

$$B(p,q)B(-p,-q) = \frac{2\pi i(p+q)}{pq}\frac{1-e^{2\pi i(p+q)}}{(1-e^{2\pi ip})(1-e^{2\pi iq})},$$
$$\Gamma(p)\Gamma(1-p) = \frac{\pi}{\sin \pi p}, \quad \left(\int_{-\infty}^{\infty} e^{-t^2/2}dt\right)^2 = 2\pi,$$

where

$$B(p,q) := \int_0^1 t^p(1-t)^q\frac{dt}{t(1-t)}, \quad \Gamma(p) := \int_0^\infty t^p e^{-t}\frac{dt}{t}$$

are the Gamma and the Beta functions.

In this paper, we give a geometric meaning for these formulae: If one regards such an integral as the dual pairing between a (kind of) cycle and a (kind of) differential form, then the value given in the right hand side of each formula is the product of the *intersection* numbers of the two cycles and that of the two forms appeared in the left-hand side.

Of course the intersection theory is not made only to *explain* these well known formulae; for applications, see [CM], [KM], [Y1].

§2. Twisted (co)homology groups

Let $l_1, \dots, l_{n+1}$ be polynomials of degree 1 in $t_1, \dots, t_k$, $(n \geq k \geq 1)$ and $\alpha_1, \dots, \alpha_{n+1}$ be complex numbers satisfying
Assumption 1. $\alpha_j \notin \mathbb{Z}, \quad \alpha_0 := -\alpha_1 - \cdots - \alpha_{n+1} \notin \mathbb{Z}.$