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On the number of Bounding Cycles for Nonlinear Arrangements

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To Peter Orlik on his sixtieth birthday

For a real hyperplane arrangement $A \subset \mathbb{R}^n$, among the first invariants that were determined for A were the number of chambers in the complement $\mathbb{R}^n \backslash A$ by Zavslavsky [Za] and the number of bounded chambers by Crapo [Cr]. In the consideration of certain classes of hypergeometric functions, there also arise arrangements of hypersurfaces which need not be hyperplanes (see e.g. Aomoto [Ao]). In this paper we will obtain a formula for the number of bounded regions (i.e. chambers) in the complement of a nonlinear arrangement of hypersurfaces. For example, for the general position arrangements of quadrics in Figure 1, we see the number of bounded regions in the complement are respectively 1, 5, and 13.

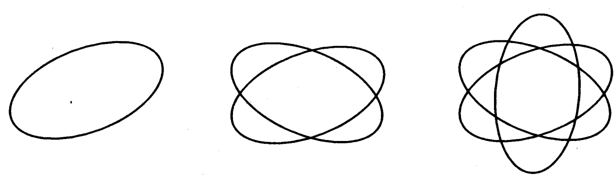

Figure 1

A computation of the number of bounded regions in the complement depends on the degrees of the hypersurfaces as well as the combinatorial structure of the arrangement. Hence, the form such a formula should take is less obvious, even given the answer for hyperplane arrangements. Moreover, in the real case for hypersurfaces of degree > 1 there is the added complication that the number depends upon the specific hypersurfaces (another choice of real quadrics could have fewer real intersections).

In the case of real hyperplane arrangements, the number of bounded regions in the complement represents an intrinsic invariant for the associated complex arrangements. Each bounded region has a bounding

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