

## Tilting Modules and their Applications

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- Summary:*
0. Introduction.
  1. Lie algebras and algebraic groups in finite characteristics.
  2. Representation of simple algebraic groups.
  3. Kempf's vanishing theorem and Weyl character formula.
  4. Good filtrations.
  5. Canonical Frobenius splittings.
  6. Tensor products of good filtrations.
  7. Tilting modules.
  8. The functor  $T^\Gamma$ .
  9. The Verlinde's Formula and the modular Verlinde's formula.
  10. Proof of the modular Verlinde's formula.
  11. Tilting modules and commutant algebras.
  12. Application of tilting modules to representation theory of  $GL(V)$ .
  13. An easy example: the fusion ring of  $SL(p-1)$ .
  14. Application of tilting modules to the symmetric group  $S_n$ .
  15. Comparison with the quantum case.
  16. Appendix: Cohomological criterion for good filtrations.

*0. Introduction:* The aim of the paper is the theory of tilting modules for a reductive algebraic group  $G$  over an algebraically closed field  $K$  of characteristic  $p$ . In order to be convenient to the reader, the paper is essentially self-contained, what explains its length. Also we tried to make it accessible to a "characteristic zero" reader. In the introduction, we will describe the content of the paper.

The first three sections contains the basic material on algebraic groups. We recall some elementary results about restricted Lie algebras (due to Jacobson [J]), the definition of Chevalley groups and the

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