

Discretely Decomposable Restrictions of Unitary Representations of Reductive Lie Groups — Examples and Conjectures

Toshiyuki Kobayashi

Table of Contents

- §1. Breaking symmetry and hidden symmetry
- §2. Analytic and algebraic notion of discretely decomposable restrictions
- §3. Compact-like actions in infinite dimensional groups
- §4. Criterion for discretely decomposable restrictions
- §5. Conjectures on discrete branching laws

§1. Breaking symmetry and hidden symmetry

1.1. The purpose of this exposition is to give a survey of the recent study of **discrete decomposable restrictions** of unitary representations of real reductive Lie groups ([24], [30], [31]). We shall also give some perspectives by examples and open problems.

There have been recent developments in connection with restrictions of unitary representations such as:

- i) finding explicit branching laws ([8], [10], [16], [23], [24], [33], [36], [37], [40], [44], [57]),
- ii) estimate of multiplicities in the branching laws ([30], [35]),
- iii) topology of modular varieties in locally Riemannian symmetric spaces ([34]),
- iv) construction of new discrete series representations for non-symmetric homogeneous spaces ([15], [22], [24], [27], [28], [32], [39]),
- v) finding explicit parameters λ for which Zuckerman-Vogan's derived functor modules $A_q(\lambda)$ are non-zero with λ singular in some special settings ([22], §4),
- vi) existence problem of compact Clifford-Klein forms of non-Riemannian homogeneous spaces ([21], [25], [26], [42]).