Advanced Studies in Pure Mathematics 26, 2000
Analysis on Homogeneous Spaces and Representation Theory of Lie Groups
pp. 77–98

K-type Structure in the Principal Series of GL_3, I

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§1. Introduction

It is about 50 years since the principal series were defined [GN], and about 45 since Harish-Chandra showed via his Subquotient Theorem [HC] that they are fundamental to understanding representations of semisimple Lie groups. However, the structure of the principal series, especially the composition structure at points of reducibility, still presents mysteries. In recent years, several authors ([HT], [Sa], [J], [Za], [L1], [L2], [HL], [Fu]), have shown that in many cases when the principal series (or, more loosely, degenerate principal series) is multiplicity-free under the action of a maximal compact subgroup, it is possible to understand their structure, including composition series and unitarity, fairly completely and explicitly. The goal of this paper is to begin an investigation of the detailed structure of some examples of principal series in which representations of the maximal compact subgroup appear with arbitrarily large multiplicities. Specifically, we investigate here the principal series of $GL_3(\mathbb{R})$. Our main finding is that each isotypic space for the maximal compact subgroup $K = 0_3$ has a unique distinguished basis compatible with the occurrence of finite-dimensional subrepresentations. Using this basis, we are able to see certain subquotients of the principal series in a manner entirely analogous to the investigations of multiplicity-free situations. In particular, in this basis we can display the full composition series when a finite-dimensional constituent occurs. However, from other considerations, we may see that at some points where the principal series is reducible, it is not at all evident from looking at our basis. This indicates a need for further investigation.

Received May 28, 1998.
Revised June 21, 1999.
[1]Research partially supported by NSF grant DMS-9622916. The author also is grateful for hospitality from the Mittag-Leffler Institute during May 1996.